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Optimization of Delivery Plan by Quantum Computing

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Quantum computing is attracting attention as a technology for solving complex combinatorial optimization problems at high speed. We have been selling delivery planning systems, one of the business support tools for logistics companies, for 20 years and have been conducting research and development of related technologies. The delivery planning systems have a function to calculate efficient delivery routes with low transportation and delivery costs, but it requires complex optimization calculations. With the aim of applying quantum computing to this optimization computation, we are formulating, implementing, and evaluating the performance required for this purpose. In this paper, we report the findings of the validation of the formulation by comparing the results obtained using the Ising machine and conventional computers.

Keywords: logistics, delivery planning, quantum computing, annealing machine

1. Introduction

The spread of online shopping has resulted in smaller delivery lots and increased logistics volume and delivery frequency.(1) In addition, the number of users of online shopping services has been surging due to a growing number of double-income families and single-person households.(2) As the competition is becoming more intense in the delivery service market, there is a growing customer need for shorter delivery times, including next-day delivery. Logistics companies need to formulate plans for the frequent delivery of large numbers of parcels. The intense competition in the delivery service market has also spurred demand for more advanced delivery services, such as same-day delivery and rapid delivery, in addition to the current next-day delivery. Meanwhile, it is also required to quickly change the initial plans and maintain the delivery service as much as possible in response to the ever-changing road conditions, such as congestion and accidents, and weather conditions, such as heavy rain and snow. If automatic delivery robots and drones are deployed for delivery in the future, it will be required to instantaneously formulate highly complicated delivery plans by taking into account various means of delivery.

In the conventional calculation for delivery planning, it takes many hours to calculate the delivery of a large number of parcels using many vehicles (for example, 100 vehicles). Thus, a delivery plan is formulated at night on the previous day and applied to deliveries on the following day. In most cases, on-site drivers must cope with changes in the delivery sequence due to congestion and disasters after the commencement of delivery. It is extremely difficult to formulate a new delivery plan for overall optimization and give instructions to drivers.

It is difficult to calculate exact solutions for combinatorial optimization problems, including the Vehicle Routing Problem, using conventional computers (hereinafter referred to as "classical computers"). Accordingly, approximate solutions are calculated within the available time by sequentially searching for solutions. In quantum computers, a kind of parallel computing is performed by using the superposition principle of quantum mechanics. Quantum computers have attracted much public attention as a technology that has the potential to calculate good solutions close to exact solutions. Meanwhile, quantum computers are still under development and incapable of solving large-scale problems due to hardware constraints. Sumitomo Electric Industries, Ltd. has long been engaged in research and development(3) on delivery planning and has marketed its delivery support package software "Haisoudesu." Based on the results and findings, we have formulated complicated delivery plans in anticipation of actual delivery using quantum computers and verified the effectiveness of the application. This paper reports the outcome.

2. Overview of Quantum Computers

The unit of information handled by quantum computers is called a qubit. Qubits enable calculation in two states of zero and one at the same time using the superposition principle in quantum mechanics. When there are n qubits, $2n$ states can be expressed at the same time. Thus, quantum computers can dramatically reduce the number of calculation steps.

Quantum computers are classified into two categories depending on the operation mode: quantum gate computers and quantum annealing computers. Quantum gate computers achieve general-purpose and high-speed processing of certain problems to ensure downward compatibility with classical computers. Meanwhile, quantum annealing computers specialize in solving combinatorial optimization problems expressed by Ising models. Target problems must be formulated in the format called Quadratic Unconstrained Binary Optimization (QUBO). Quantum annealing computers can handle more qubits compared to quantum gate computers, but even more qubits are required when solving practical combinatorial optimization problems. In this project, we conducted verification using an Ising machine capable of calculating problems formulated by QUBO at high speeds by applying conventional technologies, such as GPU and FPGA.

3. Difficulty of Combinatorial Optimization Problems

Optimization problems refer to problems in which solutions to minimize or maximize objective functions are calculated based on given constraint conditions. Depending on the format of objective functions and possible values of decision variables, optimization problems are classified into general-purpose problems, such as linear programming problems and nonlinear programming problems. Of general-purpose problems, problems in which decision variables are integers are referred to as integer programming problems, which include combinatorial optimization problems, such as knapsack problems and traveling salesman problems. Integer programming problems are considered to be NP-hard in computational complexity theory and cannot be solved in polynomial time. That is, as the scale of a problem increases, the amount of calculation increases exponentially. Exact solutions cannot be calculated in a realistic time. Thus, when solving problems of a large scale, approximate solutions are calculated in general by using general-purpose metaheuristics or techniques specialized for specific problems.

4. Target Vehicle Routing Problem

In general, logistics companies deliver parcels using multiple vehicles. They need to allocate parcels to each respective vehicle and plan the delivery sequence while taking into account various costs, including the driving distance of vehicles and on-duty hours of drivers (Fig. 1). When the number of delivery destinations is small, it is relatively easy to formulate a plan. However, with a large number of vehicles and parcels, it becomes difficult to formulate a plan manually because of the increased complexity of the plan. Delivery planning software is available as one of the tools to support the formulation of complicated delivery plans. Delivery planning software automatically formulates delivery plans to attain minimal costs by inputting various conditions, such as the number of vehicles and parcels. The software calculates optimal delivery plans by solving highly complicated optimization problems called the Vehicle Routing Problem (VRP).(4) VRP is a typical combinatorial optimization problem. Solutions in the form of more realistic delivery plans have been studied and proposed, such as Capacitated VRP (CVRP)(5) and VRP with Time Windows (VRPTW).(6),(7) Time-Dependent VRP with Time Windows (TDVRPTW), which is based on the assumption that the time required for traveling from one point to another changes depending on the time of day, was formulated by QUBO and was evaluated and verified using an Ising machine. The results are reported below.

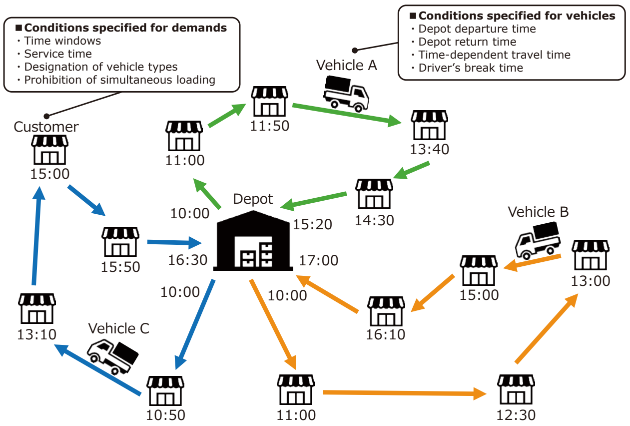

Fig. 1. Example of vehicle route plan

5. Expression of TDVRPTW by QUBO

The set of vehicles is K, the departure point of vehicle k is s_k, and the return point is e_k. The set which represents the delivery destination points of parcels is D, and the sum of sets of points which vehicle k is likely to and must visit is $P_k = D \cup \{s_k, e_k\}$. The set of time from departure from the depot to return to the depot, which is common to all the vehicles, is $T = \{1,2,\ldots,N\}$. The time in which a vehicle travels from point p to point p' at time t and becomes ready to travel from point p', namely, the total time including the traveling time from point p to point p' at time t, break time during traveling, and waiting time and work time after arrival at point p', is $c_{p,p'}^{t}$, which is a non-negative integer. Next, $q_{t,p}^{k}$, a binary variable that serves as the decision variable in the formulation, is defined as follows:

$$q_{t,p}^{k} \coloneqq \begin{cases} 1 : \text{vehicle } k \text{ departs from point } p \text{ at time } t \\ 0 : \text{otherwise} \end{cases}$$

Based on the above, when TDVRPTW is expressed by QUBO, energy function $\mathcal{H}$ can be expressed by the following Eq. (1):

$$\mathcal{H} = \sum_{i=1}^{9} w_i \mathcal{H}_i \quad \cdots\cdots (1)$$

where w_i is a weighting coefficient for each term, $\mathcal{H}_i$ is a term that expresses objective functions and constraint conditions for optimization, which consist of Eq. (2) to Eq. (10) below.

$$\mathcal{H}_1 = \sum_{k\in K}\sum_{t\in T} t\left(q_{t,e_k}^{k} - q_{t,s_k}^{k}\right) \quad \cdots\cdots (2)$$

$$\mathcal{H}_2 = -\sum_{k\in K}\sum_{t\in T}\sum_{d\in D} q_{t,d}^{k} \quad \cdots\cdots (3)$$

$$\mathcal{H}_3 = \sum_{k\in K}\sum_{t_1\in T}\sum_{p_1\in P_k\setminus\{e_k\}} \left(q_{t_1,p_1}^{k} \sum_{p_2\in P_k\setminus\{p_1,s_k\}} \sum_{t_2=t_1}^{t_1+c_{p_1,p_2}^{t_1}-1} q_{t_2,p_2}^{k} \right) \cdots (4)$$

$$\mathcal{H}_4 = \sum_{k_1 \in K} \sum_{t_1 \in T} \sum_{d \in D} \left(q^{k_1}_{t_1,d} \sum_{\substack{k_2 \in K \\ k_2 \geq k_1}} \sum_{t_2 \in T \setminus \{t_1\}} q^{k_2}_{t_2,d} \right) \quad \cdots\cdots (5)$$

$$\mathcal{H}_5 = \sum_{k_1 \in K} \sum_{t \in T} \sum_{d \in D} \left(q^{k_1}_{t,d} \sum_{\substack{k_2 \in K \\ k_2 > k_1}} q^{k_2}_{t,d} \right) \quad \cdots\cdots (6)$$

$$\mathcal{H}_6 = \sum_{k \in K} \sum_{t_1 \in T} \left(q^{k}_{t_1,s_k} \sum_{t_2 = 1}^{t_1 - 1} \sum_{p \in P_k} q^{k}_{t_2,p} \right) \quad \cdots\cdots (7)$$

$$\mathcal{H}_7 = \sum_{k \in K} \sum_{t_1 \in T} \left(q^{k}_{t_1,e_k} \sum_{t_2 = t_1 + 1}^{N} \sum_{p \in P_k} q^{k}_{t_2,p} \right) \quad \cdots\cdots (8)$$

$$\mathcal{H}_8 = \sum_{k \in K} \left(\sum_{t \in T} q^{k}_{t,s_k} - 1 \right)^2 \quad \cdots\cdots (9)$$

$$\mathcal{H}_9 = \sum_{k \in K} \left(\sum_{t \in T} q^{k}_{t,e_k} - 1 \right)^2 \quad \cdots\cdots (10)$$

$\mathcal{H}_1$ and $\mathcal{H}_2$ are objective functions. $\mathcal{H}_1$ means minimization of the total operation time of the respective vehicles from the departure point to the return point, and $\mathcal{H}_2$ means maximization of the number of parcels delivered. That is, these terms express the delivery of as many parcels as possible in the shortest time possible. $\mathcal{H}_3$ to $\mathcal{H}_9$ correspond to constraint conditions. Each term is 0 when constraints are met and a positive value when constraints are not met to provide a penalty. $\mathcal{H}_3$ means the prohibition of traveling of a vehicle from one point to another in less than the given time required, and $\mathcal{H}_4$ means that the number of vehicles which visit each delivery destination point is one or less and once or less. However, $\mathcal{H}_4$ does not express the violation of different vehicles visiting each point at the same time. Thus, a penalty is provided in $\mathcal{H}_5$. $\mathcal{H}_6$ and $\mathcal{H}_7$ prohibit vehicles from visiting delivery destination points before departure from the depot and after returning to the depot. $\mathcal{H}_8$ and $\mathcal{H}_9$ are terms expressing that vehicles must depart from the depot and return to the depot once each.

6. Reduction in Decision Variables

In QUBO expressed by Eq. (1), the number of decision variables is $O(|K||D|N)$, which is derived from multiplying the number of vehicles by the number of points by the number of times. Thus, the calculation requires an enormous number of qubits. However, solutions can be partly calculated in advance due to the characteristics of TDVRPTW. The number of qubits required can be reduced by assigning 0 to the decision variables and fixing the values in advance. Here, variables focusing on the traveling time from a delivery destination point to a return point are fixed, and variables related to the break time specified to drivers are fixed.

In the problem setting in this project, exceedance of the time for vehicles to return to the depot from N was considered a constraint violation, and the traveling time from one point to another was non-negative. It is evident that, when a vehicle is ready to depart from a point, the time to directly return to the depot at that time must be N or less. The time to return to the depot for all time/point pairs can be calculated in advance. Thus, variables are fixed in accordance with the following Eq. (11):

$$q^{k}_{t,p} = 0 \text{ if } t + c^{t}_{p,e_k} > N \ (k \in K, t \in T, p \in P_k) \quad \cdots\cdots (11)$$

When the break time of a driver is specified, it is evident that the vehicle must not start traveling from the point during the break. When the break time of a driver is specified in section (t_s, t_e), variables are fixed based on the following Eq. (12):

$$q^{k}_{t,p} = 0 \ (k \in K, t_s \leq t < t_e, p \in P_k) \quad \cdots\cdots (12)$$

7. Evaluation Using an Ising Machine and the Results

An evaluation was made using the sample data of TDVRPTW to confirm the validity of the formulated QUBO and compare the performance with that of a classical computer. For the evaluation, 12 types of Vehicle Routing Problems with different numbers of vehicles and parcels delivered were prepared, and the results of solutions using a classical computer and an Ising machine were compared for the respective types of problems. A vehicle routing engine developed by Sumitomo Electric was used as the software for operation on a classical computer. Meanwhile, an Annealing Engine (AE) of Fixstars Amplify Corporation was used as an Ising machine. A calculation environment of the basic plan was used for the evaluation. Amplify AE of v0.7.0 and Amplify SDK of v0.9.1 were used.

The evaluation results are shown in Table 1, which presents the results when the annealing time of 10 seconds was specified. The error rate shows the values based on the comparison between the formulation results using a classical computer and the total operation time of vehicles.

Table 1 shows that the calculation results were equivalent to those of a classical computer for problems involving up to nine vehicles and 54 parcels.

Table 1. Problem sizes and evaluation results using an Ising machine

Number of vehicles	Number of parcels	Number of spins	Total operation time	Error rate
1	6	151	23	±0%
2	11	448	45	±0%
3	17	948	70	±0%
4	23	1,529	97	±0%
5	29	2,425	121	±0%
6	36	3,552	149	±0%
7	42	4,746	176	±0%
8	48	6,072	204	±0%
9	54	7,632	229	±0%
10	60	9,410	259	+1.6%
11	66	11,319	296	+5.3%
12	71	13,188	323	+5.6%

When the size of a problem was larger than that of these problems, the total operation time of vehicles became longer and the delivery efficiency decreased compared to those of a classical computer. This is considered to be attributable to the fact that the structure of the energy function represented by Eq. (1) becomes complicated as the size of the problem increases, resulting in insufficient searching for solutions during the annealing time of 10 seconds, which was specified in this project. In fact, the total number of terms of constraint conditions represented by Eq. (3) to Eq. (10) increases exponentially: about 20,000 for one vehicle and six parcels, about three million for six vehicles and 36 parcels, and about 20 million for 12 vehicles and 71 parcels. Thus, to formulate delivery plans more efficiently using an Ising machine, it is necessary to make arrangements such as expressing constraint conditions using QUBO as plain as possible and ensuring a long annealing time.

8. Conclusion

This paper presented the performance evaluation results using an Ising machine with TDVRPTW expressed by QUBO. In this evaluation, we confirmed that results equivalent to those of a classical computer were obtained for problems of a small scale involving up to nine vehicles and 54 parcels and that formulation by QUBO was correct. We will formulate delivery plans by factoring in more complicated conditions and develop techniques to obtain optimal solutions for large-scale problems.

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- Haisoudesu is a trademark or registered trademark of Sumitomo Electric Industries, Ltd.

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